Drag Relationships for Liquid Droplets Settling in a Continuous Liquid

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Experiments are reported in which the drag of single liquid droplets settling in a tall column of another lighter immiscible liquid are measured. The experimental data for the eight pairs of liquids that were tested covered a range of droplet Reynolds numbers from 10^{-3} to 10^4. Two regimes of droplet settling were encountered. In the first regime, the droplets remained spherical, and the drag agreed very well with established solid sphere drag models. In the second regime, the droplets became deformed and oscillated; the drag was found to depart suddenly from predictions of spherical models and to increase with increasing Reynolds number. Empirical models for the point of departure from spherical drag and the coefficient of drag in the unstable regime are derived.

Introduction

To determine the configuration of multifluid pools with bubbling-induced mass entrainment across the liquid-liquid interface, it is necessary to address the motion of the entrained liquid drops as they resettle downwards under the action of gravity. Without mass entrainment across the two-fluid interface by the rising gas bubbles, the pool would remain stratified. If the bubbles can entrain the lower liquid in their wakes into the upper liquid layer and if the rate of settling of the entrained drops can balance the rate of entrainment driven by the bubbles, the upper pool could take on an equilibrium mixture. If the rate of settling cannot balance the rate of entrainment, the lower fluid would continue to be pumped into the upper layer by the rising bubbles until the pool would become fully mixed. There have been a number of studies of the relationship between the terminal velocity and drag coefficient of spheres falling in a semi-infinite medium. The broad scope of applications for such research includes the analysis of particle motion in fluids, the study of phenomena such as rainfall, boiling and aerosol decontamination, the design of condensing economizers and air pollution control equipment, as well as the design of evaporative coolers and cooling towers. We will discuss the existing data and data from our studies of liquid droplets over a wide range of Reynolds and Weber numbers and compare this to the predictions of existing drag correlations. In addition, the effect of droplet deformation and oscillation upon the droplet drag will be discussed.

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Background

The construction of a force balance on a sphere of diameter d moving at a constant terminal velocity, u, results in the following expression for the drag coefficient,

$$C_d = \frac{4d\,g}{3u_d^2}\left(\frac{\rho_d - \rho_l}{\rho_l}\right) \tag{1}$$

which is usually presented as a function of the Reynolds number, Re.

A number of investigators have examined the drag of liquid droplets falling in liquid-filled test columns. Hu and Kintner (1955) studied droplets of ten different organic fluids falling in a square, water-filled column. The study covered a Reynolds number range of $10-2{,}000$. The authors observed that there was little evidence of surface flow or internal circulation effects in the droplets below a Reynolds number of 300, since the C_d vs. Re curve in this range was nearly identical to that for solid spheres. Beyond this Reynolds number range, droplet behavior departed from the predictions of solid sphere theory. A peak diameter corresponding to the maximum droplet velocity was observed in each system studied, and it was found that droplet drag increased rapidly if the Reynolds number were to exceed the critical Reynolds number based on this peak diameter and velocity. The critical Weber number based on these values for peak diameter and velocity was found to be $We_{crit}=3.58$. They hypothesized that droplet oscillation induced at this peak velocity was the primary cause for this rapid increase in droplet drag. The authors proposed an empirical dimensionless physical property group,

$$P=\frac{3}{4}\frac{Re^4}{C_d We^3} \tag{2}$$

and found that the data from all but one of their fluid systems collapsed to one curve when presented in the following format,

$$[C_d \cdot We \cdot P^{0.15}] \text{ vs. } [Re/P^{0.15}] \tag{3}$$

Winnikow and Chao (1966) performed a similar study examining droplets of four organic liquids over a Reynolds number range of 100 to 1,000. The experiments were performed in a 4-in.^{2} (2,580 mm^2) water-filled test column. The authors identified three distinct regions in the drag coefficient vs. Reynolds number relationship. For Reynolds numbers less than 1.0, the liquid droplets were observed to maintain their spherical shape and the data were found to be in excellent agreement with the Stokes solution for solid spheres. A transition region was indicated for $80<Re<300$, in which droplet deformation occurred and the measured drag coefficients were higher or lower (depending on the particular fluid involved) than those for solid spheres of equivalent diameter. A third region, beginning at Reynolds numbers between 700 to 900, marked the onset of droplet oscillation and a sharp increase in droplet drag with a slight increase in the Reynolds number. The authors noted that the minimum drag and maximum droplet terminal velocity coincided with this onset of oscillation, as had been found by Hu and Kintner (1955). They hypothesized that this increase in droplet drag was due to increased pressure drag resulting from droplet deformation and a shift in the position of boundary layer separation. The continued increase in droplet drag was attributed to the combined effect of deformation-induced form drag and droplet oscillation. They proposed an average critical Weber number of 4.08 as an indication of this onset of liquid droplet oscillation.

Beard and Pruppacher (1969) studied the drag on water droplets injected into a wind tunnel with a hypodermic needle. The air speed in the tunnel was adjusted until droplets of a certain size became suspended. Droplet diameters were determined by photographic analysis. The authors' study covered a Reynolds number range of 0.2 to 200, and they proposed the following correlations for determining droplet drag as a function of Reynolds number,

$$\frac{D}{D_s}=\begin{cases}1+0.102\,Re^{0.955}, & 0.2<Re<2.0\\ 1+0.115\,Re^{0.802}, & 2.0<Re<21.0\\ 1+0.189\,Re^{0.632}, & 21.0<Re<200.0\end{cases} \tag{4}$$

where D is the measured droplet drag and D_s is the Stokes drag for a solid sphere at the same Reynolds number. The results agreed quite well with those of prior studies involving both solid spheres and liquid droplets over a comparable Reynolds number range. For Reynolds numbers greater than 200, the droplet drag was observed to increase due to distortions from the spherical shape. As a result these equations were not recommended for Reynolds numbers greater than 200 nor for the case of droplets which are experiencing distortion or oscillation.

Klee and Treybal (1956) examined both rising and falling droplets for eleven fluids in a square test column. The study covered a Reynolds number range from one to 2,000. The authors identified two distinct regions in plotting the droplet velocity vs. diameter, and noted that the maximum point in each curve corresponded to the point at which droplet oscillation developed. Droplet drag coefficients were consistently found to be smaller than the values predicted for solid spheres, a phenomenon which the authors attributed to the oscillation and internal circulation present in a droplet. The interfacial tension between the dispersed and continuous phase liquids was found to be a controlling parameter for droplet stability. The authors presented the following equations relating Re, C_d, and We,

$$Re=22.2\,C_d^{-5.18}\,We^{-0.169} \quad \text{(Region I)} \tag{5}$$

$$Re=0.00418\,C_d^{2.91}\,We^{-1.81} \quad \text{(Region II)} \tag{6}$$

where I and II refer to their regions of observed droplet stability and oscillation, respectively. The critical Weber number at which the droplets deviated from solid sphere drag was found to be, on the average, approximately equal to 8.21. This result was in reasonable agreement with previous authors. The range of the critical Reynolds number over which the droplet drag coefficient was observed to deviate from solid sphere data was small, in the range 300–600.

Similar studies were performed by Licht and Narasimhamurty (1955) and by Krishna et al. (1969), who studied liquid droplets of six fluids and 31 fluids, respectively. The droplets were of various organic liquids and the continuous fluid was once again water, just as in the previous studies. In both these studies, the measured droplet drag coefficient was observed to conform closely to the established theory for solid spheres at low Reynolds numbers. Departure from solid sphere drag behavior once again occurred in a narrow range of the droplet Reynolds number; the critical Reynolds number reported for the drag deviation was in the range 600–1,000 for Licht (1955) and 500–1,000 for Krishna (1969). For Krishna's data, the critical Weber number for the departure of drag from solid sphere drag (an average of 31 data points) was found to be 4.04. Further examination suggests that this good agreement, particularly between the results of Hu and Kintner (1955), Winnikow and Chao (1966), and Krishna et al. (1969) may be a result of the narrow range of variation in physical properties of the fluids used in these studies, and may only be applicable to organic liquid drops settling in water.

Experiment and Procedures

In the present study, droplet settling experiments were conducted with eight different pairs of fluids in order to further examine the droplet drag-Reynolds number functional relationship. Liquids utilized for the droplet phase included water, water/$CuSO_4$, bromoform, and mercury; liquids utilized for the continuous phase included 10 and 100 cs. silicone oils, hexane, and water. The fluid pairs were chosen in an attempt to encompass the widest range possible for the appropriate dimensionless scaling parameters. The parameters that are most important are the droplet Reynolds number, Weber number, and drag coefficient. The droplet volume was varied over the range 1 μL to 6 mL; combined with the variations in liquid properties, this resulted in a database for the droplet drag coefficient covering seven orders of magnitude in Reynolds number, from 10^{-3} to 10^4.

Liquid droplets of precise volumes were created by use of an Eppendorf digital pipette. The droplet volume was calibrated by depositing a number of the same volume droplets into a calibrated burette; the independent measurement of the sum of the volumes as well as the weight was accepted as verification. Once the droplet volume was verified, a droplet was deposited in a Teflon holding cup which was installed at the top of the test column and submerged in the continuous fluid. To begin the test, the holding cup was rotated on its axis and inverted, allowing the droplet to free fall downward. An initial fall distance was allowed in order that the droplet entered the measured course at its terminal velocity. If the droplet did not leave the holding cup smoothly and in one piece, that test was rejected. The descent of the droplet was either electronically or manually timed as it traversed a fixed course (1 m) inscribed on the column's outer surface. The times of flight for twenty separate trials for each droplet volume were then averaged, and this average time was used to calculate an average terminal velocity over the measured distance. Precise measurements were made of the physical properties for each of the fluids studied over a temperature range appropriate to the droplet settling data (20–40°C). Temperature-dependent values of density, interfacial tension, and viscosity were used to calculate the Reynolds number, Weber number, and the droplet drag coefficient.

Experimental Results

The experimentally measured drag coefficient data for four of the droplet systems are illustrated in Figure 1. These droplets were observed to remain spherical during the fall period, and the measured drag coefficients were found to agree well with established models for solid sphere drag; shown in Figure 1 are the Stokes flow model for Reynolds numbers less than 0.1 and the correlations of Beard and Pruppacher (1969) for Reynolds numbers in the range 0.2 to 200 (Eq. 4), shown here extrapolated to $Re = 1{,}000$. These data are in excellent agreement with the droplet drag data of Hu and Kintner (1955), Licht and Narasimhamurty (1955), and Krishna et al. (1969) for Reynolds numbers below the values of the critical Reynolds number for droplet oscillation observed in each of those studies.

Previous studies have shown that, at the onset of droplet oscillation, the droplet drag suddenly departs from the solid sphere drag curve (this will be called the critical drag coefficient) and increases rapidly with a further increase in the Reynolds number. This Reynolds number has been called the critical Reynolds number and was measured by previous investigators to lie in the range 500–1,000. For nearly all these data, the corresponding critical Weber number for the onset of this phenomenon was measured to be approximately equal to four.

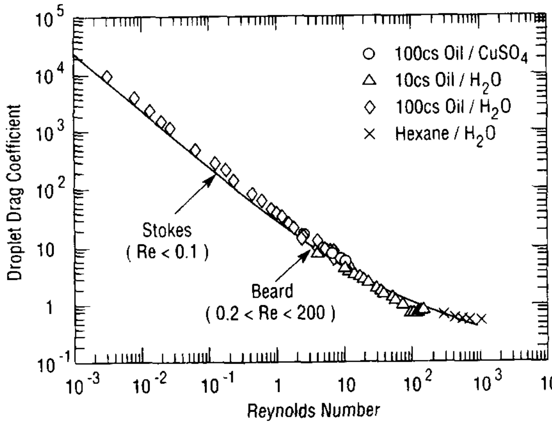

Figure 1. Drag coefficients vs. Reynolds number for stable liquid drops settling in a liquid column.

The data for seven series of experiments from the present study are shown in Figure 2. The data for five of the seven fluid pairs followed the stable drag curve of Beard and Pruppacher (1969) over a range of Reynolds numbers of 0.2–1,000. However, depending upon the fluid pair tested, deviation from the Beard model was observed to suddenly begin at Reynolds numbers as low as eight and as high as 1,000. As shown in Figure 2, six of the seven fluid pairs tested experienced this departure from stable drag. Interestingly, no droplet system tested in the present study, nor from any of the previous studies cited, were able to exceed a Reynolds number of 1,000 without experiencing drag departure from the stable drag curve.

In the present study, the critical Weber number was found to cover a much wider range than previously reported, a range of 2–50. An empirical relationship between the critical droplet Reynolds number and critical Weber number for the onset of droplet oscillation and departure from stable drag is shown in Figure 3. It is apparent from this figure that nearly all the previous data experienced drag enhancement at a Weber number of four and in a narrow range of Reynolds number. The present data, however, demonstrated this behavior for Weber numbers over the range 2–50 and Reynolds numbers from less

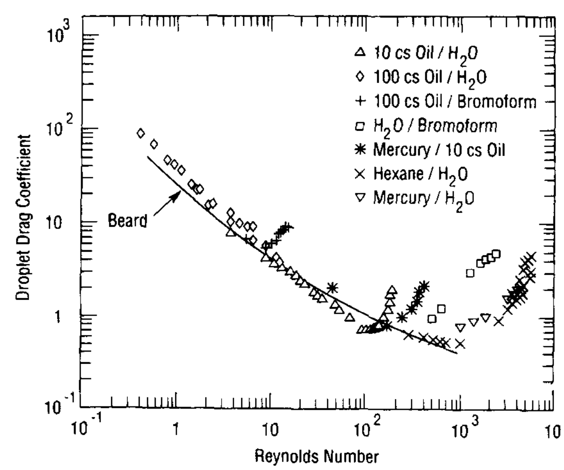

Figure 2. Effect of oscillation and instability on the droplet drag coefficient.

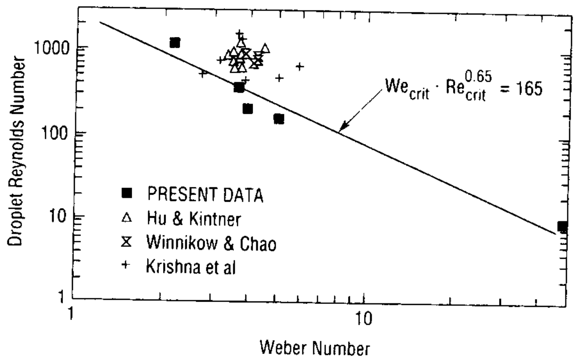

Figure 3. Critical droplet Reynolds number vs. critical Weber number for onset of droplet oscillation and deviation from stable drag.

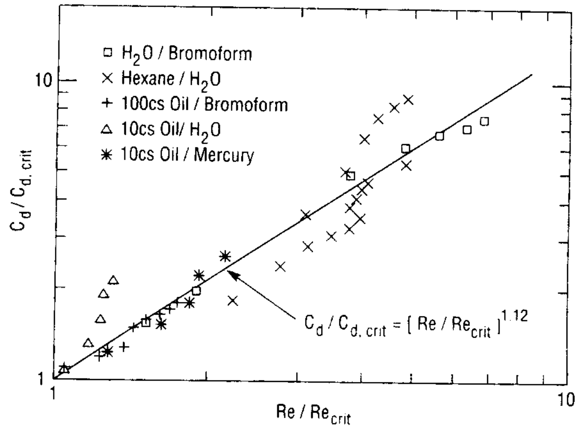

Figure 4. Correlation of unstable drag data.

than ten to greater than 1,000. The numerical values for the critical droplet Reynolds numbers and the critical droplet Weber numbers for each of the five data points shown in Figure 3 are listed in Table 1. From the present data, a functional relationship between the Reynolds number and Weber number at this point of departure from stable drag was calculated as shown below.

$$We_{\rm crit} \cdot Re_{\rm crit}^{0.65} = 165 \tag{7}$$

It is clear that for values of $We \cdot Re^{0.65}$ less than 165, a particular droplet-continuous liquid pair would be stable, and the droplet drag would be well characterized by any of the solid sphere drag models. As the critical value of $We \cdot Re^{0.65}$ is approached, the droplet would begin to experience oscillation and deformation. For values greater than 165, droplet oscillation and deformation would result in an increase in the droplet drag far above that for an equivalent spherical drop as the Reynolds number were to increase.

Liquid droplets for which the value of $We \cdot Re^{0.65}$ exceeds 165 will have deviated from the stable drag curve and the trend of the droplet drag coefficient will be to increase with increasing Reynolds number. The data from the present study that demonstrated this behavior as shown in Figure 2 are those that depart from the Beard line and move up and to the right. As shown in the figure, this behavior was found to occur for Reynolds numbers as low as eight. It is evident that many droplet systems may experience this drag enhancement and that for practical applications, the unstable drag regime, described by $We \cdot Re^{0.65}$ greater than 165, may be the dominant regime. In order to develop an empirical model for the data in this regime (for $We \cdot Re^{0.65} > 165$), the droplet drag coefficients and droplet Reynolds numbers for these data were normalized by the corresponding critical values for departure from stable drag which were previously determined and shown in Figure 3. From the normalized data, a functional empirical relationship between the droplet drag coefficients and Reynolds numbers was calculated by least-squares regression analysis and is given by,

$$C_d / C_{d,\rm crit} = [\mathrm{Re}/\mathrm{Re}_{\rm crit}]^{1.12} \tag{8}$$

The normalized data are shown along with Eq. 8 in Figure 4. This is the drag relationship that is recommended for droplet systems for which $We \cdot Re^{0.65}$ exceeds 165.

Table 1. Observed Critical Reynolds and Weber Numbers for Onset of Deviation from Stable Drag

Droplet Continuous Fluid	$Re_{\rm crit}$	$We_{\rm crit}$
Water droplet in Hexane	1,154.4	2.17
Water droplet in 10 cs. Silicone Oil	151.7	5.03
Bromoform droplet in Water	346.1	3.67
Mercury droplet in 10 cs. Silicone Oil	198.5	3.90
Bromoform droplet in 100 cs. Silicone Oil	8.8	47.82

Conclusions

Data for droplet drag have been found to agree well with established solid sphere or spherical droplet models and correlations as long as the droplets maintained their spherical shape while settling under the influence of gravity. At the Reynolds and Weber numbers indicative of the onset of droplet oscillation and deformation, the drag coefficients for the droplets were observed to suddenly depart from the solid sphere drag curve and increase with increasing Reynolds number. This sudden departure from classical drag has been found to occur for Reynolds and Weber numbers over the ranges 10–1,000 and 2–50, respectively. No stable drag data were found for Reynolds number greater than approximately 1,000. It was found that the parameter, $We \cdot Re^{0.65}$, at this critical point was approximately 165. For values less than this criterion, the droplet drag was stable and agreed well with spherical drag models; for values greater than this criterion, the droplet drag departed from stable drag models and the drag coefficient increased with increasing Reynolds number.

When the droplet drag coefficients and Reynolds numbers in the unstable drag regime were normalized by the corresponding critical values for the departure from stable drag, the normalized droplet drag coefficient was found to correlate well to the normalized droplet Reynolds number.

Notation

C_d = drag coefficient

d = equivalent spherical diameter
D = droplet drag
g = gravitational acceleration
Re = Reynolds number
u = velocity
We = Weber number

Greek letters

ρ = density
σ = interfacial tension
ν = kinematic viscosity

Subscripts

d = droplet
l = liquid
s = Stokes
crit = critical transition value

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